

Relating the cosmological constant and slow roll to conformal symmetry breaking

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Abstract: We show that a theory with conformal invariance, which is explicitly broken by small terms, provides a solution to the fine tuning problem of the cosmological constant. In the absence of the symmetry breaking terms, the cosmological constant is zero. Its value in the full theory is controlled by the symmetry breaking terms. The symmetry breaking terms also provide the slow roll conditions, which may be useful in constructing a model of inflation.

1 Introduction

It has been argued that if conformal invariance is broken by a soft mechanism then it might be possible to preserve its consequences even in the full quantum theory [1–6]. This allows the possibility of constructing a theory in which the cosmological constant can be set identically to zero. Let us consider the mechanism proposed in [2]. We consider a toy model with two real scalar fields. The Lagrangian density, in four dimensions, may be written as,

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_M + \mathcal{L}_{SB} \quad (1)$$

where \mathcal{L}_G and \mathcal{L}_M are the gravity and matter Lagrangian

$$\mathcal{L}_G = -(\beta_1^2 \chi^2 + \beta_2^2 \phi^2)R \quad (2)$$

$$\mathcal{L}_M = \frac{1}{2} [(\partial\chi)^2 + (\partial\phi)^2] - \lambda(\phi^2 - \lambda_1^2 \chi^2)^2 \quad (3)$$

The \mathcal{L}_{SB} breaks conformal invariance explicitly and was not included in [2]. We shall specify it below. We notice that \mathcal{L}_M does not include all the terms allowed by conformal invariance. It is possible to write down one more term quartic in the fields, which has been set to zero. As explained in [2], this is necessary to break scale invariance spontaneously. In the full quantum theory this is needed in order to have a well defined perturbative expansion.

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The model as specified above has a conformal anomaly and hence breaks scale invariance. Within the framework of dimensional regularization this is traced to the fact that the couplings, λ and λ_1 , are not dimensionless when $d \neq 4$. However it is possible to generalize the action such that it maintains conformal invariance in d dimensions. Let us define the field ω such that [2],

$$\omega^2 \equiv (\beta_1^2 \chi^2 + \beta_2^2 \phi^2) \quad (4)$$

In $d = 4 - \epsilon$ dimensions, we can make all terms in the action conformally invariant by multiplying them with a suitable power of ω . In particular the potential term gets modified to,

$$(\phi^2 - \lambda_1^2 \chi^2)^2 \rightarrow (\phi^2 - \lambda_1^2 \chi^2)^2 (\omega^2)^{-\delta} \quad (5)$$

where $\delta = (d - 4)/(d - 2) = -\epsilon/(2 - \epsilon)$. The scalar field kinetic energy terms as well as the term proportional to R remains unchanged. In $d \neq 4$, the potential terms will involve fractional powers of the field. These terms are handled by expanding the fields around their classical values. For example, let χ_0 and ϕ_0 represent the classical values of the fields χ and ϕ respectively and $\hat{\chi}$ and $\hat{\phi}$ represent the corresponding quantum fluctuations around the classical solution. Hence we can express χ and ϕ as,

$$\begin{aligned} \chi &= \chi_0 + \hat{\chi} \\ \phi &= \phi_0 + \hat{\phi} \end{aligned} \quad (6)$$

As long as $\chi_0 \neq 0$ and $\phi_0 \neq 0$, quantum expansion is well defined. Actually we only require the classical value, ω_0 , of the field, ω , defined in Eq. 4, to be non-zero. Hence a necessary condition for a consistent perturbative expansion in this theory is that $\omega_0 \neq 0$. This procedure is called the GR-SI prescription in [2].

The classical values of the scalar fields, χ_0 and ϕ_0 , generate all the dimensional parameters in the theory, such as, the gravitational constant, the electroweak scale, the Higgs mass etc. As we shall see later, $\phi_0 = \lambda_1 \chi_0$. Making a quantum expansion, we find that the mass terms of the scalar fields are given by,

$$\mathcal{L} = -4\lambda\lambda_1^2\chi_0^2(\hat{\phi} - \lambda_1\hat{\chi})^2 \quad (7)$$

Hence the field, $(\hat{\phi} - \lambda_1\hat{\chi})$ becomes massive. We shall choose the parameter range such that $\lambda_1 \ll 1$. Hence the massive field is dominantly equal to $\hat{\phi}$. The orthogonal combination, proportional to, $(\hat{\chi} + \lambda_1\hat{\phi})$, remains massless. This field is dominantly $\hat{\chi}$.

Following the procedure described above, [2] show that the standard predictions of conformal invariance are preserved by the theory. In particular, the theory predicts a massless dilaton, at all orders in the perturbation theory. Despite the presence of conformal invariance, the theory does predict running coupling constant. At one loop, [2, 7] also argue that the Higgs mass is stable under quantum corrections. In the toy model under consideration, the Higgs field is identified with the field ϕ . However it has been argued that this problem does not really get solved since, in the presence of the Planck scale and the electroweak scale, the theory requires some very small parameters, which have to be fine tuned at each order [8].

One has to impose some constraints on the parameter values in order that the perturbation theory remains well defined. At all orders in perturbation theory, one has to impose a constraint on parameters, such that conformal invariance is spontaneously broken. If this is not preserved then the perturbation theory does not make sense. Once this condition is imposed, the theory predicts a massless dilaton in this theory.

Another important point is that removal of all divergences might require terms of the kind, ϕ^6/χ^2 . Such terms are allowed by scale invariance. Hence the perturbation theory may be more complicated in these theories, requiring large number of parameters [9, 10]. Due to the presence of such terms, the theory is not renormalizable. Hence it loses predictive power at mass scale above Planck mass. This is not a very serious problem since the additional terms are suppressed by Planck mass. Furthermore it may be related to the non-renormalizability of gravity. This is because the scalar fields, which might lead to such terms, are intrinsically tied to gravity. For example, the classical values of the fields, χ and ϕ , generate the gravitational constant, G . In any case, even in the presence of such terms, the consequences of conformal invariance remain valid. The theory also predicts zero cosmological constant. The reason is that it has no dimensional parameter. Hence the effective potential, at any order in perturbation theory contains terms which are quartic in fields, multiplied by a function of the ratio of the fields, such as $r = \phi/\chi$. We may express the effective potential as, $V_{eff} = \chi^4 U(r)$ [2]. We shall assume that V_{eff} is such that, in the absence of symmetry breaking terms, it is minimized for χ_0 and ϕ_0 not equal to 0 or $\pm\infty$. The minimization conditions then imply,

$$\begin{aligned} \left. \frac{dU(r)}{dr} \right|_{r=r_0} &= 0 \\ U(r_0) &= 0 \end{aligned} \quad (8)$$

where $r_0 = \frac{\phi_0}{\chi_0}$. The one loop effective potential has been explicitly constructed in [2]. After imposing the conditions, Eq. 8, it is found that conformal symmetry is spontaneously broken at this order also. By conformal invariance, the potential displays degenerate minima, such that χ_0 and ϕ_0 take a continuous range of values and $r_0 = \chi_0/\phi_0$ remains fixed. Eq. 8 also implies that,

$$V_{eff}(\chi_0, \phi_0) = 0 \quad (9)$$

and hence leads to zero cosmological constant.

One may be concerned that the constraints, Eq. 8, might themselves require fine tuning [8] of parameters. However this does not arise, in the following sense. Consider the potential of the model,

$$V(\phi, \chi) = \lambda(\phi^2 - \lambda_1^2 \chi^2)^2 + \lambda_2 \chi^4 \quad (10)$$

where we have included all the possible terms that can arise in the potential, consistent with conformal invariance. The minimization conditions, Eq. 8, can be satisfied only if,

$$\lambda_2 = 0 \quad (11)$$

This is not fine tuning in the sense that we do not need to maintain a very small value of λ_2 . We may compare this with the standard problem of fine tuning of the cosmological constant [11,12]. The problem is most severe if we have to fine tune the cosmological constant at each order to a very small value. If we can set the cosmological constant identically to zero, even if there is no symmetry demanding this, then this is not as severe a problem. Of course, ideally it would be elegant if a symmetry or some other mechanism may demand a vanishing cosmological constant. However in the absence of such a mechanism, it would still represent progress if at each order in perturbation theory we don't need to fine tune the cosmological constant to a very small value and can simply set it to zero. In the present case also no symmetry requires Eq. 11. However we do need to impose this constraint in order that perturbation theory is well defined. Furthermore, it is satisfying that we do not need to fine tune λ_2 to a very small value.

We point out that there is currently considerable effort to study the potential implications of conformal invariance in cosmology or high energy physics. Several model being studied are based on local conformal invariance [13–22]. The implications of global scale invariance has also been investigated [23–28].

2 Explicit conformal symmetry breaking

We next add a small conformal symmetry breaking term in the action. This term is of the form,

$$\mathcal{L}_{SB} = -\frac{1}{2}m_1^2\chi^2 - \frac{1}{2}m_2^2\phi^2 - \Lambda + \dots \quad (12)$$

Here m_1 and m_2 are the mass terms of the two fields and Λ a cosmological constant. For aesthetic reasons, we may choose to set $\Lambda = 0$, but the theory does not require it. As long as these terms are zero, such terms cannot be generated, at any order in perturbation theory, by the action which is symmetric under conformal transformations. Hence we can choose Λ , m_1 and m_2 to be arbitrarily small without any fine tuning. Let us first set $\Lambda = 0$. Its effect will be discussed later. The basic point is that the mass terms lift the degeneracy in the potential. The location of the global minimum depends on the choice of symmetry breaking terms. We point out that by a suitable choice of such terms, the global minimum might arise at non-zero values of χ and ϕ . At any particular time the fields may take values such that the potential is not at its minimum. Hence it will produce an effective cosmological constant. The fields will also evolve slowly, as assumed in several models of inflation [29] or dark energy [30]. The slow roll is now controlled by the small symmetry breaking part of the action.

We display this mechanism by a choosing simple model. We set $m_2 \approx 0$. We choose the parameters β_1 and β_2 to be small compared to unity. These parameters need not be very small and hence do not require acute fine tuning. In the absence of symmetry breaking terms, the potential is minimized for

$$\phi_0 = \lambda_1 \chi_0 \quad (13)$$

We shall assume that $\lambda_1 \ll 1$ and hence $\phi_0 \ll \chi_0$. As we shall see, we require $\chi_0 \gg M_{PL}$, such that $\beta_1 \chi_0 \approx M_{PL}$. The value of λ_1 need not be very small, since ϕ_0 may be of order Planck

or GUT scale. Alternatively, ϕ_0 might be of the order of electroweak scale. In this case λ_1 does require acute fine tuning, which is related to the standard problem of maintaining a low Higgs mass in the presence of a very large mass scale in the scalar potential. This problem is not solved in this model [2, 8]. In the full theory, including symmetry breaking terms, Eq. 13 will not yield the true minimum of the potential.

Let us first work directly in the Jordon frame and, for simplicity, just ignore the term proportional to R . As we shall, for our choice of parameters, we get the same result in Einstein frame. We are interested in solving the scalar field equations of motion in order to determine the effective cosmological constant. The equations of motion, ignoring space derivatives are given by,

$$\begin{aligned}\ddot{\phi} + 3H\dot{\phi} + 4\lambda\phi(\phi^2 - \lambda_1^2\chi^2) &= 0 \\ \ddot{\chi} + 3H\dot{\chi} - 4\lambda\lambda_1^2\chi(\phi^2 - \lambda_1^2\chi^2) + m_1^2\chi &= 0\end{aligned}\tag{14}$$

where H is the Hubble parameter. Assuming an approximate solution of the form, Eq. 13, we find that $\dot{\phi} \approx 0$. Here we set the second derivatives of the fields equal to zero, since they likely to be more suppressed in comparison to the first derivatives. We also find that,

$$\dot{\chi} \sim m_1^2 \frac{\chi_0}{H}\tag{15}$$

For slow roll conditions to be satisfied, we require

$$\dot{\chi}^2 << m_1^2\chi_0^2\tag{16}$$

which implies that,

$$m_1 << H\tag{17}$$

Hence for slow roll, we require that the symmetry breaking terms are much smaller than the Hubble parameter. Such small terms would normally require acute fine tuning. However in the present case these are protected by conformal symmetry.

The solution leads to vacuum energy equal to $m_1^2\chi_0^2/2$. Hence, in order that it generates a sufficiently large value of the effective cosmological constant, we require,

$$m_1^2\chi_0^2 \sim M_{PL}^2 H^2\tag{18}$$

In the Jordon frame, the gravitational constant undergoes a slow evolution, which has been ignored in the above equation. This evolution can be consistently ignored as long as the slow roll conditions are satisfied. It is useful to perform the entire calculation in the Einstein frame which, as we shall see below, leads to the same result. Eq. 18, along with the slow roll condition leads to the constraint,

$$\chi_0 >> M_{PL}\tag{19}$$

We point out that the model contains some small parameters, such as, β_1 and λ_1 , which are not protected by conformal invariance. However these parameters need not be very small.

Their precise values depend on the model under consideration. For our purposes these may be of the order of 10^{-3} . The possibility that λ_1 may be very small and its associated fine tuning has already been discussed above. The important point is that the mass parameters, such as, m_1 , are extremely tiny in comparison to other mass scales, such as M_{PL} , ϕ_0 etc. Their small value, however, is protected against quantum corrections by conformal invariance.

We next perform the calculation in the Einstein frame. We make the conformal transformation, such that,

$$g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu} \quad (20)$$

where

$$\Omega^2 \equiv \frac{M_{PL}^2}{\omega^2} \quad (21)$$

The Lagrangian density in terms of the transformed variables can be written as,

$$\begin{aligned} \mathcal{L}_E &= -\frac{M_{PL}^2}{16\pi} R - \frac{3M_{PL}^2}{8\pi} g^{\mu\nu} \partial_\mu \ln \Omega \partial_\nu \ln \Omega \\ &+ \frac{\Omega^2}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{\Omega^2}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \Omega^4 V \end{aligned} \quad (22)$$

where V is the potential,

$$V = \lambda(\phi^2 - \lambda_1^2 \chi^2)^2 + \frac{1}{2} m_1^2 \chi^2 + \frac{1}{2} m_2^2 \phi^2 \quad (23)$$

where, as before, we shall assume, $m_2 \approx 0$. We next obtain the equations of motion keeping only the time derivative. Using the slow roll approximation we drop the second derivatives. Under this approximation, we obtain,

$$\begin{aligned} -3H\dot{\phi} + \frac{9H}{4\pi} \frac{\beta_2^2 \phi}{\omega^2} (\beta_1^2 \chi \dot{\chi} + \beta_2^2 \phi \dot{\phi}) - \frac{M_{PL}^2}{\omega^2} \frac{\partial V}{\partial \phi} + \frac{4M_{PL}^2}{\omega^4} \beta_2^2 \phi V &= 0 \\ -3H\dot{\chi} + \frac{9H}{4\pi} \frac{\beta_1^2 \chi}{\omega^2} (\beta_1^2 \chi \dot{\chi} + \beta_2^2 \phi \dot{\phi}) - \frac{M_{PL}^2}{\omega^2} \frac{\partial V}{\partial \chi} + \frac{4M_{PL}^2}{\omega^4} \beta_1^2 \chi V &= 0 \end{aligned} \quad (24)$$

In the absence of symmetry breaking terms we again find the same result as Eq. 13, with $\dot{\phi} = 0$ and $\dot{\chi} = 0$. Solving the full equations, assuming the relationship Eq. 13, between classical values of ϕ and χ , we find that both $\dot{\phi}$ and $\dot{\chi}$ are related to the symmetry breaking terms. The second terms on the left hand side of both the equations are negligible since, $\beta_1 \ll 1$ and $\beta_2 \ll 1$. Given that, at leading order, $\omega \sim \beta_1 \chi_0 \sim M_{PL}$, we again find that $\dot{\chi}$ is given by Eq. 15. In the present case $\dot{\phi} \neq 0$. However it is clear that $\dot{\phi} \ll \dot{\chi}$, being suppressed by the factor $\beta_2^2 \phi_0 / (\beta_1^2 \chi_0)$. Hence we again get exactly the same condition, Eq. 19, as obtained in the Jordan frame.

2.1 Higher Orders

The above analysis may be performed at any order in perturbation theory using the effective potential. While computing the quantum contributions to effective potential, we ignore the

symmetry breaking terms. The symmetry breaking terms are assumed to be extremely small and hence are expected to give negligible contributions at higher orders to the effective potential. Hence the effective potential at any order can be expressed as,

$$V_{eff} = \chi^4 U(r) + \frac{1}{2} m_1^2 \chi^2 + \frac{1}{2} m_2^2 \phi^2 \quad (25)$$

where, the term $\chi^4 U(r)$ is obtained entirely from the symmetry preserving part of the action. We require that, in the absence of symmetry breaking terms, at each order the effective potential displays a minimum, where its value is nonzero and finite. Hence we have to impose some conditions on the counter terms so that this holds [2]. Due to conformal invariance this minimum value of the potential can only be zero.

We now replace V in Eq. 24 by V_{eff} . We are interested in a solution subject to the conditions specified by Eq. 8. Imposing these conditions in Eq. 24, we find that, $\dot{\phi}$ and $\dot{\chi}$ are both proportional to symmetry breaking terms. The value of $\dot{\chi}$ is again given by Eq. 15 and $\dot{\phi} \ll \dot{\chi}$. Hence we can maintain their small values without any fine tuning.

2.2 Non-zero cosmological constant

We next discuss the case where the symmetry breaking terms contain a non-zero cosmological constant, Λ . In this case we set the masses, m_1 and m_2 , equal to zero. This case is very simple. The degeneracy of the minimum does not get lifted, i.e. the minimum is exactly degenerate even when we include symmetry breaking terms. Hence the equations of motion satisfy Eq. 13 exactly. The theory now has non-zero cosmological constant. However it does not receive large corrections from the symmetry preserving terms. At higher orders also Eq. 13 is maintained by a suitable choice of counter terms [2]. Hence we still have a degenerate minima, with the minimum value approximately equal to zero, up to the corrections due to symmetry breaking term, Λ .

3 Applications to inflation and dark energy

The mechanism that we have discussed above may be applied either to inflation or to dark energy. Let us first discuss the case of inflation. In this case it is simplest to choose symmetry breaking terms such that $\Lambda = 0$. We can choose the mass terms, m_1 and m_2 , sufficiently small to satisfy the slow roll conditions. Inflation ends when the fields reach the true minima of the potential. The phenomenon acts like the standard large field inflation [29]. The symmetry breaking terms have to be of the order of the inflationary scale. Hence this theory will have conformal breaking of the order of inflationary scale and will not solve the fine tuning problem of dark energy. However the inflationary slow roll condition can be met without any fine tuning.

Alternatively we may accommodate inflation by fine tuning the symmetry preserving terms and the symmetry breaking terms may be only of the order of dark energy. In this case we may either introduce an explicit cosmological constant or masses, m_1 and m_2 . In case of cosmological constant, the constraint on the field χ , Eq. 19, is not applicable. However this constraint is

applicable if dark energy is generated by the masses, which leads to a slow evolution of the fields.

4 Conclusions

In this paper we have shown that conformal symmetry provides a mechanism which partially alleviates the problem of fine tuning of the cosmological constant. We use the GR-SI prescription in which the conformal invariance can be maintained in the full quantum theory [2]. However the perturbation theory gets more complicated and renormalizability of the theory does not remain maintained [9, 10]. Hence the theory loses predictability beyond a certain mass scale, which in the present model is taken to be the Planck scale. Hence this absence of renormalizability is not a very serious issue at low energies. The conformal invariance in the theory is spontaneously broken for a certain range of parameters. The perturbation theory makes sense only if this can be accomplished. Hence we have to impose an additional constraint on the theory, not required by conformal invariance. We have argued that this constraint does not amount to fine tuning of a parameter since it does not involve maintaining a small value of a parameter at each order in perturbation theory. It simply requires setting some parameter value identically to zero. Given this constraint, the perturbation theory can be well defined.

If we impose exact conformal invariance on the theory, then it predicts zero cosmological constant. We introduce small conformal symmetry breaking terms. These involve mass terms of scalar fields and/or explicit cosmological constant. Since the symmetry preserving part does not generate such terms at any order in the perturbation theory, we can maintain their small value without any fine tuning. We may identify the cosmological constant with dark energy. Alternatively the scalar mass terms lead to slowly rolling scalar field and hence can also generate dark energy. Another possibility is that the model may be applied to generate inflation. Detailed application of the model to dark energy or inflation is not pursued in this paper.

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